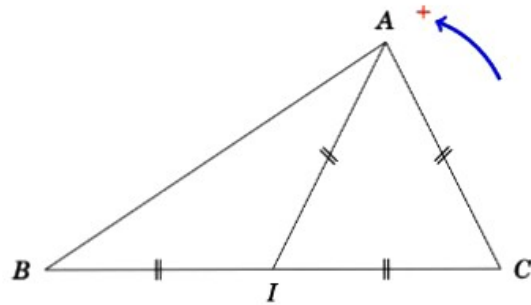


تمرين 1

أعطي القيس الرئيسي للزوايا التالية (حسب الشكل) :
 $(\vec{AC}, \vec{AB})$ $(\vec{BI}, \vec{BA})$ $(\vec{IB}, \vec{IA})$ $(\vec{CA}, \vec{CI})$



تمرين 2

بين أن x و y هما قيسين لنفس الزاوية الموجهة :
 $x = 123\pi$, $y = \frac{115\pi}{2}$, $y = \frac{729\pi}{6}$; $x = \frac{11\pi}{4}$, $y = \frac{-5\pi}{4}$
 $x = \frac{-3\pi}{2}$, $y = \frac{\pi}{2}$; 3π

تمرين 3

عين في كل حالة من الحالات التالية القيس الرئيسي للزاوية التي قياسها
 $\alpha = \frac{14\pi}{3}$; $\alpha = \frac{35\pi}{2}$; $\alpha = \frac{721\pi}{5}$; $\alpha = \frac{2012\pi}{3}$
 على الدائرة المثلثية :

تمرين 4

في المستوي الموجه لدينا : $(\vec{U}; \vec{V}) = \frac{\pi}{3}$
 عين قيساً لكل زاوية من الزوايا الموجهة التالية : $(3\vec{V}; 2\vec{U})$ $(\vec{V}; \vec{U})$ $(-\vec{V}; -\vec{U})$ $(-3\vec{V}; 7\vec{U})$ $(2\vec{V}; -3\vec{U})$

تمرين 5

لتكن (C) الدائرة المثلثية مرفقة بمعلم متعامد ومتجانس $(O, \vec{i}, \vec{j})$
 لتكن A و B نقطتين من (C) حيث : $(\vec{OI}, \vec{OA}) = \frac{\pi}{6}$
 $(\vec{OI}, \vec{OB}) = \frac{3\pi}{4}$
 عين قيساً للزوايا الموجهة : $(\vec{OJ}, \vec{OB})$ $(\vec{OJ}, \vec{OA})$ $(\vec{OA}, \vec{OB})$

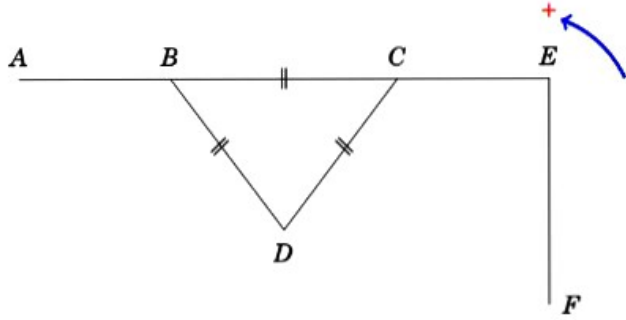
تمرين 6

DCB مثلث قائم في D و لتكن A نقطة من قطعة المستقيم [DB] ،
 حيث : $AC = AB = 2\text{cm}$
 ولدينا : $(\vec{AB}, \vec{AC}) = \frac{5\pi}{6} + 2k\pi$ $k \in \mathbb{Z}$

(1) أنشئ الشكل .
 (2) أحسب كل من $(\vec{AC}, \vec{CB})$; $(\vec{CA}, \vec{CB})$; $(\vec{BC}, \vec{BA})$
 (3) أحسب BC ، DA ، DB ، DC ، ثم إستنتج $\sin \frac{\pi}{12}$ و $\cos \frac{\pi}{12}$

تمرين 7

عين قيساً للزوايا الموجهة التالية :
 $(\vec{AB}, \vec{BD})$; $(\vec{CE}, \vec{CD})$; $(\vec{CE}, \vec{EF})$; $(\vec{BD}, \vec{EF})$



تمرين 8

لدينا : $\cos x = \frac{1}{2}$
 (1) أحسب $\sin(\pi - x)$; $\cos(\frac{\pi}{2} - x)$; $\sin(\frac{\pi}{2} - x)$; $\sin x$
 (2) أحسب $\tan(\pi - x)$; $\tan(\frac{\pi}{2} - x)$; $\tan x$

تمرين 9

أحسب A ، B و C حيث :
 $A = \cos(\frac{\pi}{2} + x) + \sin(\pi - x) + \cos(\frac{\pi}{2} - x) - \sin(-x)$
 $B = \cos(\pi + x) + \cos(\frac{3\pi}{2} + x) + \sin(3\pi - x) + \sin(\frac{5\pi}{2} - x)$
 $C = \cos(\frac{7\pi}{2} + x) + \cos(\frac{7\pi}{2} - x) + \cos(\frac{\pi}{2} + x) - \sin(\frac{\pi}{2} - x)$

تمرين 10

حل في المجال $[0; 2\pi]$ المعادلات التالية :
 $\cos -2x = -\frac{\sqrt{3}}{2}$, $\cos 3x = -\frac{\sqrt{2}}{2}$, $\cos x = \frac{\sqrt{3}}{2}$
 $\sin -2x = -\frac{\sqrt{3}}{2}$, $\sin 3x = -\frac{\sqrt{2}}{2}$, $\sin(x + \frac{\pi}{3}) = \frac{\sqrt{3}}{2}$
 $\cos x + \sin x = 0$, $\cos(x - \frac{\pi}{4}) = \sin(3x + \frac{\pi}{2})$
 $\sin^2 x - \sin x - 6 = 0$,

تمرين 11

حل في $\mathbb{R}$ المعادلات التالية :
 $\sin 3x = \cos(x - \frac{\pi}{4})$, $\sin 2x = \cos(x + \frac{\pi}{6})$,
 $2\cos^2 x - \sin^2 x - \sin x - 6 = 0$, $\sin(2x + \frac{\pi}{6}) = \cos(x + \frac{\pi}{3})$
 $4\cos^2 x + 2(1 - \sqrt{3})\cos x - \sqrt{3} = 0$, $3\cos x - 2 = 0$

تمرين 12

حل في المجموعة $[0; 2\pi]$ المتراجحات التالية :
 $2\cos 2x - \sqrt{3} \geq 0$, $\sqrt{2}\cos 3x + 1 \leq 0$, $2\cos x < 1$
 $2\sin 5x + \sqrt{3} \geq 0$, $\sqrt{2}\sin 4x - 1 \leq 0$, $2 < 1$, $\cos 4x - \frac{1}{2} \geq 0$
 $2\sin 4x - \sqrt{2} > 0$

$$(\vec{OJ}, \vec{OB}) = (\vec{OJ}, \vec{OI'}) + (\vec{OI'}, \vec{OB})$$

$$= \frac{\pi}{2} - (\vec{OB}, \vec{OI'})$$

$$= \frac{\pi}{2} - \frac{\pi}{4}$$

$$(\vec{OJ}, \vec{OB}) = \frac{\pi}{4}$$

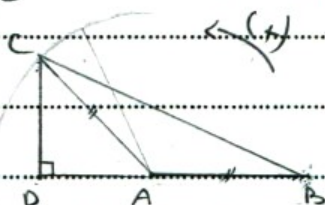
$$(\vec{OA}, \vec{OB}) = (\vec{OA}, \vec{OJ}) + (\vec{OJ}, \vec{OB})$$

$$= -(\vec{OJ}, \vec{OA}) + \frac{\pi}{4}$$

$$= \frac{\pi}{3} + \frac{\pi}{4}$$

$$(\vec{OA}, \vec{OB}) = \frac{7\pi}{12}$$

$$(\vec{AB}, \vec{AC}) = \frac{5\pi}{6}$$



(ج) حساب اقياس الزوايا

$$\bullet (\vec{BC}, \vec{BA}) = (\pi - \frac{5\pi}{6}) \times \frac{1}{2}$$

$$(\vec{BC}, \vec{BA}) = \frac{\pi}{12}$$

$$\bullet (\vec{CA}, \vec{CB}) = (\vec{BC}, \vec{BA}) = \frac{\pi}{12}$$

لان المثلث ABC متساوي الساقين

$$\bullet (\vec{AC}, \vec{CB}) = (-\vec{CA}, \vec{CB})$$

$$= (\vec{CA}, \vec{CB}) + \pi$$

$$= \frac{\pi}{12} + \frac{\pi}{12} = \frac{13\pi}{12}$$

$$3) \frac{35\pi}{2} = \frac{34\pi}{2} + \frac{\pi}{2} = 2(17)\frac{\pi}{2} + \frac{\pi}{2}$$

وهذه القيس الرئيسية α هو $\frac{\pi}{2}$

$$4) \frac{14\pi}{3} = \frac{12\pi}{3} + \frac{2\pi}{3} = 4\pi + \frac{2\pi}{3}$$

وهذه القيس الرئيسية α هو $\frac{2\pi}{3}$

الفتيل (سجل)

تمرين 4

$$(\vec{u}, \vec{e}) = \frac{\pi}{3}$$

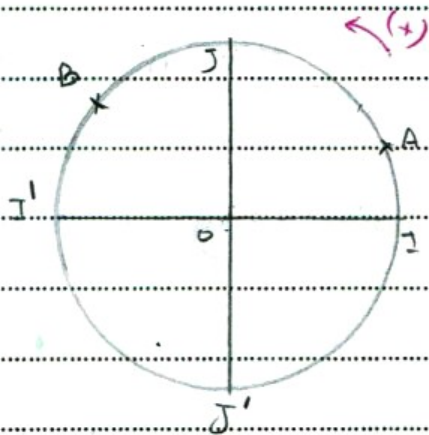
$$\bullet (\vec{v}, \vec{u}) = -(\vec{u}, \vec{e}) = -\frac{\pi}{3}$$

$$\bullet (3\vec{e}, 2\vec{u}) = (\vec{e}, \vec{u}) = -\frac{\pi}{3}$$

$$\bullet (2\vec{e}, -3\vec{u}) = (\vec{e}, \vec{u}) + \pi = -\frac{\pi}{3} + \pi = \frac{2\pi}{3}$$

$$\bullet (-3\vec{v}, 7\vec{u}) = (\vec{e}, \vec{u}) + \pi = \frac{2\pi}{3}$$

$$\bullet (-\vec{e}, -\vec{u}) = (\vec{e}, \vec{u}) = -\frac{\pi}{3}$$



تمرين 5

$$(\vec{OI}, \vec{OA}) = \frac{\pi}{6} ; (\vec{OI}, \vec{OB}) = \frac{3\pi}{4}$$

$$(\vec{OJ}, \vec{OA}) = -(\vec{OA}, \vec{OJ})$$

$$= -(\vec{OA}, \vec{OI}) - (\vec{OI}, \vec{OJ})$$

$$= (\vec{OI}, \vec{OA}) - (\vec{OI}, \vec{OJ})$$

$$= \frac{\pi}{6} - \frac{\pi}{2} = -\frac{2\pi}{6}$$

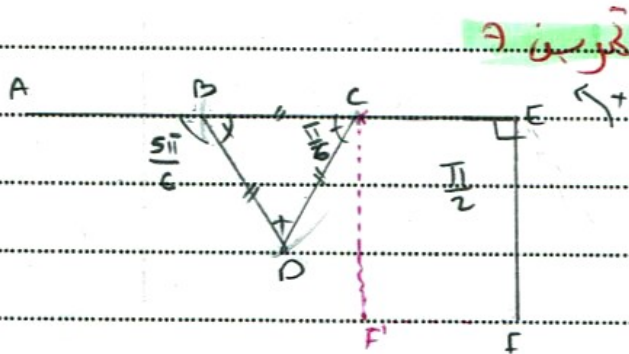
$$(\vec{OJ}, \vec{OA}) = -\frac{\pi}{3}$$

$$\sin(\vec{BD}, \vec{BA}) = \sin(\vec{BC}, \vec{BD})$$

$$= \frac{1}{2\sqrt{2+\sqrt{3}}} = \frac{\sqrt{2+\sqrt{3}}}{4+2\sqrt{3}}$$

$$\cos(\vec{BC}, \vec{BA}) = \cos(\vec{BC}, \vec{BD})$$

$$= \frac{-2+\sqrt{3}}{2\sqrt{2+\sqrt{3}}}$$



$$(\vec{BD}, \vec{EF}) = (\vec{BD}, \vec{BC}) + (\vec{BC}, \vec{EF})$$

$$= (\vec{BD}, \vec{BC}) + (\vec{BC}, \vec{CE})$$

$$= (\vec{BD}, \vec{BC}) + (-\vec{CB}, \vec{CE})$$

$$= (\vec{BD}, \vec{BC}) + (\vec{CB}, \vec{CE}) + \pi$$

$$= \frac{\pi}{6} + \frac{\pi}{2} + \pi$$

$$(\vec{BD}, \vec{EF}) = \frac{10\pi}{6} = \frac{5\pi}{3}$$

$$(\vec{CE}, \vec{EF}) = (-\vec{EC}, \vec{EF})$$

$$= (\vec{EC}, \vec{EF}) + \pi$$

$$= \frac{\pi}{2} + \pi = \frac{3\pi}{2}$$

$$(\vec{CE}, \vec{CD}) = -(\vec{CD}, \vec{CE}) = \pi - \frac{\pi}{6}$$

$$= -\pi + \frac{\pi}{6} = -\frac{5\pi}{6}$$

(3) حساب الأطوال

DC

لدينا المثلث ACD قائم في D و

$$(\vec{AC}, \vec{AD}) = \frac{\pi}{6}$$

و حيث

$$(\vec{AC}, \vec{AD}) = \frac{DC}{AC}$$

وعليه

$$DC = \sin(\vec{AC}, \vec{AD}) \times AC$$

$$= \frac{1}{2} \times 2$$

أي

$$DC = 1$$

DA

$$\cos(\vec{AC}, \vec{AD}) = \frac{DA}{AC}$$

أي

$$DA = \cos(\vec{AC}, \vec{AD}) \times AC$$

$$= \frac{\sqrt{3}}{2} \times 2$$

$$DA = \sqrt{3}$$

أي

DB

$$DB = DA + AB = \sqrt{3} + 2$$

BC

حساب قياسا جواريا

$$BC = \sqrt{DB^2 + DC^2}$$

$$= \sqrt{(2+\sqrt{3})^2 + 1^2}$$

$$= \sqrt{4+3+4\sqrt{3}+1}$$

$$BC = \sqrt{8+4\sqrt{3}}$$

$$BC = 2\sqrt{2+\sqrt{3}}$$

$$\sin \frac{\pi}{12}$$

$$= \frac{+\frac{\sqrt{3}}{2}}{-\frac{1}{2}} = +\sqrt{3}$$

مربعین ۹

$$\begin{aligned} (\vec{AB}, \vec{BD}) &= (\vec{PA}, \vec{BD}) + \pi \\ &= \frac{5\pi}{6} + \pi = \frac{11\pi}{6} \end{aligned}$$

$$\cos x = 1/2$$

مربعین ۸

$$\begin{aligned} A &= \cos\left(\frac{\pi}{2} + x\right) + \sin(\pi - x) \\ &\quad + \cos\left(\frac{\pi}{2} - x\right) - \sin(x) \end{aligned}$$

$$= -\sin x + \sin x + \sin x + \sin x$$

$$A = 2\sin x$$

$$\begin{aligned} B &= -\cos x + \cos\left(\pi + \frac{\pi}{2} + x\right) \\ &\quad + \sin(2\pi + \pi - x) + \sin\left(2\pi + \frac{\pi}{2} - x\right) \end{aligned}$$

$$\begin{aligned} &= -\cos x - \cos\left(\frac{\pi}{2} + x\right) \\ &\quad + \sin(\pi - x) + \sin\left(\frac{\pi}{2} - x\right) \end{aligned}$$

$$\begin{aligned} &= -\cos x + \sin x + \sin x \\ &\quad + \cos x \end{aligned}$$

$$B = 2\sin x$$

$$\begin{aligned} C &= \cos\left(4\pi - \frac{\pi}{2} + x\right) + \cos\left(4\pi - \frac{\pi}{2} - x\right) \\ &\quad + \sin x - \cos x \end{aligned}$$

$$= \cos\left(-\frac{\pi}{2} + x\right) + \cos\left(-\frac{\pi}{2} - x\right)$$

$$= \sin x - \cos x$$

$$= \cos\left(-\left(\frac{\pi}{2} - x\right)\right) + \cos\left(-\left(\frac{\pi}{2} + x\right)\right)$$

$$= \sin x - \cos x$$

$$= \sin x - \sin x - \sin x - \cos x$$

$$C = -\sin x - \cos x$$

$$\sin x = \frac{1}{2}$$

پایه

$$\cos^2 x + \sin^2 x = 1$$

$$\frac{1}{4} + \sin^2 x = 1$$

$$\sin^2 x = 1 - \frac{1}{4}$$

$$\sin^2 x = \frac{3}{4}$$

اصول

$$\sin x = -\frac{\sqrt{3}}{2} \text{ و } \sin x = +\frac{\sqrt{3}}{2}$$

$$\sin\left(\frac{\pi}{2} - x\right) = \cos x = \frac{1}{2}$$

$$\cos\left(\frac{\pi}{2} - x\right) = \sin x = +\frac{\sqrt{3}}{2}$$

$$\sin(\pi - x) = \sin x = +\frac{\sqrt{3}}{2}$$

$$\cos(\pi - x) = -\cos x = -\frac{1}{2}$$

$$\tan x = \frac{\sin x}{\cos x} = +\sqrt{3}$$

$$\tan\left(\frac{\pi}{2} - x\right) = \frac{\sin\left(\frac{\pi}{2} - x\right)}{\cos\left(\frac{\pi}{2} - x\right)}$$

$$= \frac{1/2}{+\sqrt{3}/2} = +\frac{1}{\sqrt{3}}$$

$$\tan(\pi - x) = \frac{\sin(\pi - x)}{\cos(\pi - x)}$$

وهم المعادلات تصبح

$$\cos(2x) = \cos\left(\frac{3\pi}{6}\right)$$

نكافي

$$\begin{cases} -2x = \frac{3\pi}{6} + 2k\pi \\ -2x = -\frac{3\pi}{6} + 2k\pi \end{cases}$$

$$\begin{cases} x = -\frac{3\pi}{12} - k\pi \\ x = \frac{3\pi}{12} - k\pi \end{cases}$$

$$\bullet \sin\left(x + \frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$$

نظم أ ب

$$\sin\left(x + \frac{\pi}{3}\right) = \sin\frac{\pi}{3}$$

نكافي

$$\begin{cases} x + \frac{\pi}{3} = \frac{\pi}{3} + 2k\pi \\ x + \frac{\pi}{3} = \pi - \frac{\pi}{3} + 2k\pi \end{cases}$$

$$\begin{cases} x = 0 + 2k\pi \\ x = -\frac{2\pi}{3} + 2k\pi \end{cases}$$

$$\bullet \cos\left(x - \frac{\pi}{4}\right) = \sin\left(3x + \frac{\pi}{2}\right)$$

نقوم بحل واحد الطرفين

$$\cos x = \sin\left(\frac{\pi}{2} - x\right)$$

نظم أ ب

$$\begin{aligned} \cos\left(x - \frac{\pi}{4}\right) &= \sin\left(\frac{\pi}{2} - x + \frac{\pi}{4}\right) \\ &= \sin\left(-x + \frac{3\pi}{4}\right) \end{aligned}$$

تمرين 10

حل المعادلات

$$\bullet \cos x = \frac{\sqrt{3}}{2}$$

$$\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$$

نظم أ ب

$$\cos x = \cos\frac{\pi}{6}$$

$$\begin{cases} x = \frac{\pi}{6} + 2k\pi \\ x = -\frac{\pi}{6} + 2k\pi \end{cases}$$

نكافي

$$\bullet \cos x = -\frac{\sqrt{2}}{2}$$

$$\cos\frac{3\pi}{4} = -\frac{\sqrt{2}}{2}$$

نظم أ ب

$$\cos(\pi - x) = \cos(\pi - x) = -\cos x$$

$$\cos\left(\pi + \frac{\pi}{4}\right) = -\cos\frac{\pi}{4} = -\frac{\sqrt{2}}{2}$$

وهم المعادلات تصبح

$$\cos x = \cos\frac{5\pi}{4}$$

$$\cos x = \cos\frac{5\pi}{4}$$

نكافي

$$\begin{cases} x = \frac{5\pi}{4} + 2k\pi \\ x = -\frac{5\pi}{4} + 2k\pi \end{cases}$$

$$\cos(2x) = -\frac{\sqrt{3}}{2}$$

$$\cos\left(\pi + \frac{\pi}{6}\right) = -\frac{\sqrt{3}}{2}$$

نظم أ ب

$$\sin^2 x - \sin x - 6 = 0$$

$$X = \sin x \quad \text{بفرض}$$

المعادلة تصبح

$$X^2 - X - 6 = 0$$

$$\Delta = b^2 - 4ac$$

$$\Delta = 1 + 24$$

$$\Delta = 25$$

المعادلة التربيعية لها حلان

$$x_2 = \frac{1-5}{2}$$

$$x_2 = -1$$

$$x_1 = \frac{1+5}{2}$$

$$x_1 = 3$$

$$\sin x = X$$

$$\sin x = 3 \quad \text{لأن } X = 3 \quad \text{لا}$$

لأنه لا يوجد

$$\sin x = -1 \quad \text{أي } X = -1 \quad \text{نعم}$$

$$\sin x = \sin -\frac{\pi}{2} = -1 \quad \text{نعم}$$

المعادلة تصبح

$$\sin x = \sin -\frac{\pi}{2}$$

بذلك

$$\begin{cases} x = -\frac{\pi}{2} + 2k\pi \\ x = \pi + \frac{\pi}{2} + 2k\pi \end{cases}$$

المعادلة التربيعية لها حلان

$$S = \left\{ -\frac{\pi}{2} + 2k\pi, \frac{3\pi}{2} + 2k\pi \right\}$$

وعلى المعادلة الثانية

$$\sin(-x + \frac{3\pi}{4}) = \sin(3x + \frac{\pi}{2})$$

بذلك

$$\begin{cases} -x + \frac{3\pi}{4} = 3x + \frac{\pi}{2} + 2k\pi \\ -x + \frac{3\pi}{4} = \pi - 3x - \frac{\pi}{2} + 2k\pi \end{cases}$$

$$\begin{cases} -4x = \frac{\pi}{2} - \frac{3\pi}{4} + 2k\pi \\ x = \pi - \frac{\pi}{2} - \frac{3\pi}{4} + 2k\pi \end{cases}$$

$$\begin{cases} x = \frac{\pi}{16} + \frac{1}{2}k\pi \\ x = -\frac{\pi}{4} + k\pi \end{cases}$$

$$\sin x + \cos x = 0$$

$$\therefore \sqrt{1^2 + 1^2} = \sqrt{2}$$

$$\frac{1}{\sqrt{2}} \sin x + \frac{1}{\sqrt{2}} \cos x = 0$$

$$\frac{\sqrt{2}}{2} \sin x + \frac{\sqrt{2}}{2} \cos x = 0$$

$$\sin \frac{\pi}{4} \sin x + \cos \frac{\pi}{4} \cos x = 0$$

$$\cos(\frac{\pi}{4} - x) = 0$$

بذلك

$$\cos(\frac{\pi}{4} - x) = \cos \frac{\pi}{2}$$

نحل المعادلة

مقربین 12 $[0, 2\pi[$

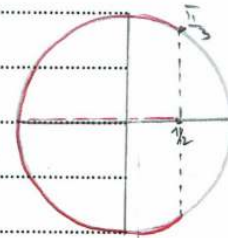
$$\bullet 2 \cos x < 1$$

$$\cos x < \frac{1}{2}$$

ای

$$\cos x < \cos \frac{\pi}{3}$$

$$x \in \left[\frac{\pi}{3}, \frac{5\pi}{3} \right]$$



$$\bullet \sqrt{2} \cos 3x + 1 \leq 0$$

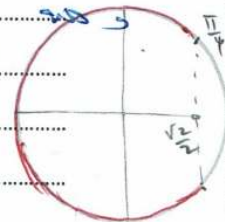
$$\cos 3x \leq \frac{1}{\sqrt{2}}$$

$$\cos 3x \leq \frac{\sqrt{2}}{2}$$

$$\cos 3x \leq \cos \frac{\pi}{4} \quad \text{و}$$

$$3x \in \left[\frac{\pi}{4}, \frac{7\pi}{4} \right]$$

$$x \in \left[\frac{\pi}{12}, \frac{7\pi}{12} \right]$$



$$\bullet 2 \cos 2x - \sqrt{3} > 0$$

$$\cos 2x > \frac{\sqrt{3}}{2}$$

$$\cos 2x > \cos \frac{\pi}{6}$$

و

$$2x \in \left[0, \frac{\pi}{6} \right] \cup \left[\frac{11\pi}{6}, 2\pi \right]$$

و

$$x \in \left[0, \frac{\pi}{12} \right] \cup \left[\frac{11\pi}{12}, \pi \right]$$

