

جديد



مجلة Maths

50

نهاية معقدة

مرفقة بالحل

المستوى



النهائي و الجامعي

جمع و كتابة : شعبان أسامة

أستاذ التعليم الثانوي

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العدد الأول

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تعليق

بعد ملاحظتي لعدة منشورات في مواقع التواصل الاجتماعي تخص كيفية حل نهايات تبدو غير مالوفة و غير متداولة كثيرا. حاولت فبجمعها و ارفاقها بحلول مقترحة في هذا العمل المتواضع.

ففمنها كان عبارة عن اجتهداد شخصي و الشق الأخر هي محاولات الأساتذة (كرم مخلوف – عرفات الأسعدي) جزاهم الله كل خير

ف

فأهدي هذا العمل الى عائلتي و الى كل محب للرياضيات.

فاللهم ارحم من علمني

أ.شعبان

شملت هذه الـهـجـلـة كل الحالات السبع المعرفـة في الرياضيات

$$\begin{array}{l} \left. \begin{array}{l} \text{منهاج الثانوي} \end{array} \right\} \begin{array}{l} \frac{0}{0} \\ \frac{\infty}{\infty} \\ 0 \times \infty \\ \infty \pm \infty \\ 1^{\infty} \\ 0^0 \\ \infty^0 \end{array} \end{array}$$

بوضع: $y = \frac{1}{x} \Leftrightarrow y \rightarrow 0$

$$\begin{aligned} \lim_{y \rightarrow 0} \left(\frac{4^y + 5^y + 6^y}{3} \right)^{\frac{3}{y}} &= e^{\lim_{y \rightarrow 0} \frac{3}{y} \ln \left(\frac{4^y + 5^y + 6^y}{3} \right)} \\ &= \lim_{y \rightarrow 0} \frac{3}{y} \frac{\ln \left(\frac{4^y + 5^y + 6^y}{3} \right)}{\left(\frac{4^y + 5^y + 6^y}{3} \right) - 1} \cdot \left(\left(\frac{4^y + 5^y + 6^y}{3} \right) - 1 \right) \\ &= \lim_{y \rightarrow 0} 1 \cdot \frac{3 \left(\frac{4^y + 5^y + 6^y - 3}{3} \right)}{\left(\frac{4^y + 5^y + 6^y}{3} \right) - 1} \\ &= \lim_{y \rightarrow 0} \left(\frac{4^y - 1}{y} + \frac{5^y - 1}{y} + \frac{6^y - 1}{y} \right) \\ &= \ln 4 + \ln 5 + \ln 6 \\ &= \ln 120 \end{aligned}$$

و بالتالي: $\lim_{x \rightarrow +\infty} \left(\frac{4^{\frac{1}{x}} + 5^{\frac{1}{x}} + 6^{\frac{1}{x}}}{3} \right)^{3x} = \ln(120)$

نهاية 4: $\lim_{n \rightarrow +\infty} \left(\frac{n+9}{n+10} \right)^n$

$$\begin{aligned} \lim_{n \rightarrow +\infty} \left(\frac{n+9}{n+10} \right)^n &= e^{\lim_{n \rightarrow +\infty} n \left(\frac{n+9}{n+10} \right)} \\ &= e^{\lim_{n \rightarrow +\infty} n \frac{\left(\frac{n+9}{n+10} \right)}{\left(\frac{n+9}{n+10} \right) - 1} \cdot \left(\left(\frac{n+9}{n+10} \right) - 1 \right)} \\ &= e^{\lim_{n \rightarrow +\infty} n \cdot 1 \cdot \left(\left(\frac{n+9}{n+10} \right) - 1 \right)} \\ &= e^{\lim_{n \rightarrow +\infty} n \cdot \left(\frac{n+9-n-10}{n+10} \right)} \\ &= e^{-1} = \frac{1}{e} \end{aligned}$$

و بالتالي: $\lim_{n \rightarrow +\infty} \left(\frac{n+9}{n+10} \right)^n = \frac{1}{e}$

نهاية 5: $\lim_{x \rightarrow 1} \frac{(2x+1)^5 - 243}{(x+1)^6 - 32}$

نلاحظ أن: $x-1 = (x+1) - ((2x+1)-3) \cdot \frac{1}{2}$

نهاية 1: $\lim_{x \rightarrow 0} \frac{e^x - x - 1}{x^2}$

نضع: $x^2 = y$ نصبح:

بوضع: $\lim_{y \rightarrow 0} \frac{e^{\sqrt{y}} - \sqrt{y} - 1}{y}$

$g(y) = e^{\sqrt{y}} - \sqrt{y}$

$g'(y) = \frac{1}{2\sqrt{y}} e^{\sqrt{y}} - \frac{1}{2\sqrt{y}} = \frac{1}{2} \left(\frac{e^{\sqrt{y}} - 1}{\sqrt{y}} \right)$

أي:

$$\begin{aligned} \lim_{y \rightarrow 0} \frac{e^{\sqrt{y}} - \sqrt{y} - 1}{y} &= \lim_{y \rightarrow 0} \frac{g(y) - g(0)}{y - 0} \\ &= g'(0) = \frac{1}{2} \end{aligned}$$

لأن: $\lim_{y \rightarrow 0} \left(\frac{e^{\sqrt{y}} - 1}{\sqrt{y}} \right)$

اذن: $\lim_{x \rightarrow 0} \frac{e^x - x - 1}{x^2} = \frac{1}{2}$

نهاية 2: $\lim_{x \rightarrow 1} \frac{x\sqrt{x+3} - 2}{x^2 + x - 2}$

$$\begin{aligned} \lim_{x \rightarrow 1} \frac{x\sqrt{x+3} - 2}{x^2 + x - 2} &= \lim_{x \rightarrow 1} \frac{x(\sqrt{x+3} - \sqrt{4}) + 2(x-1)}{x^2 + x - 2} \\ &= \lim_{x \rightarrow 1} \frac{x(\sqrt{x+3} - \sqrt{4})}{(x-2)(x-1)} + \frac{2(x-1)}{(x-2)(x-1)} \\ &= \frac{3}{4} \end{aligned}$$

أي: $\lim_{x \rightarrow 1} \frac{x\sqrt{x+3} - 2}{x^2 + x - 2} = \frac{3}{4}$

نهاية 3: $\lim_{x \rightarrow +\infty} \left(\frac{4^{\frac{1}{x}} + 5^{\frac{1}{x}} + 6^{\frac{1}{x}}}{3} \right)^{3x}$

نعلم أن: $\lim_{x \rightarrow 0} \frac{a^x - 1}{x} = \ln a$

$\lim_{x \rightarrow 1} \frac{\ln x}{x-1} = 1$

$$= \lim_{x \rightarrow 1} \frac{\pi x}{(1+3x) \cdot \sin \frac{\pi}{2}(1-3x)} + \lim_{x \rightarrow 1} \frac{\frac{\pi}{2}(1-x)}{(1+3x) \cdot \sin \frac{\pi}{2}(1-3x)}$$

$$= \frac{-2}{2(4)} - \frac{0}{2(4)} = \frac{-1}{4}$$

$$\cdot \lim_{x \rightarrow 1} \frac{\sin(\pi x) + \cos\left(\frac{\pi x}{2}\right)}{(1+3x) \cdot \cos\left(\frac{3\pi x}{2}\right)} = \frac{-1}{4} \quad \text{اذن:}$$

نهاية 8: $\lim_{x \rightarrow \frac{\pi}{2}} \frac{\cos}{\cos(3x)}$

نعلم أن: $\cos(3x) = 4\cos^3(x) - 3\cos(x)$

$$\lim_{x \rightarrow \frac{\pi}{2}} \frac{\cos(x)}{\cos(3x)} = \lim_{x \rightarrow \frac{\pi}{2}} \frac{\cos(x)}{4\cos^3(x) - 3\cos(x)}$$

$$= \lim_{x \rightarrow \frac{\pi}{2}} \frac{\cos(x)}{\cos(x)(4\cos^2(x) - 3)}$$

$$= \lim_{x \rightarrow \frac{\pi}{2}} \frac{1}{(4\cos^2(x) - 3)}$$

$$= \frac{-1}{3}$$

نهاية 9: $\lim_{x \rightarrow +\infty} \sqrt[x]{x}$

نعلم أن: $\lim_{x \rightarrow 0} x \ln(x) = 0$

نضع: $y = \frac{1}{x} \Leftrightarrow y \rightarrow 0$

$$\lim_{x \rightarrow +\infty} \sqrt[x]{x} = \lim_{x \rightarrow 0^+} \left(\frac{1}{y}\right)^y = \frac{1}{\lim_{y \rightarrow 0^+} (y)^y}$$

$$\lim_{y \rightarrow 0^+} (y)^y = e^{\lim_{y \rightarrow 0^+} y \ln(y)}$$

حيث:

$$= e^0 = 1$$

و بالتالي: $\lim_{x \rightarrow +\infty} \sqrt[x]{x} = 1$

$$\lim_{x \rightarrow 1} \frac{(2x+1)^5 - 243}{(x+1)^6 - 32} = \lim_{x \rightarrow 1} \frac{2 \cdot \frac{(2x+1)^5 - 3^5}{(2x+1)^1 - 3^1}}{\frac{(x+1)^6 - 2^5}{(x+1)^1 - 2^1}} \quad \text{و منه:}$$

$$= 2 \cdot \frac{\frac{5}{6}(3)^4}{\frac{1}{1}(2)^5} = \frac{135}{32}$$

$$\cdot \lim_{x \rightarrow 1} \frac{(2x+1)^5 - 243}{(x+1)^6 - 32} = \frac{135}{32} \quad \text{اذن:}$$

نهاية 6: $\lim_{n \rightarrow +\infty} \sqrt[n]{4^n + 5^n}$

$$\lim_{n \rightarrow +\infty} \sqrt[n]{4^n + 5^n} = \lim_{n \rightarrow +\infty} (4^n + 5^n)^{\frac{1}{n}}$$

$$= \lim_{n \rightarrow +\infty} (5^n)^{\frac{1}{n}} \cdot \left(\left(\frac{4}{5} \right)^n + 1 \right)^{\frac{1}{n}}$$

$$= 5 \cdot \lim_{n \rightarrow +\infty} \left(1 + \left(\frac{4}{5} \right)^n \right)^{\frac{1}{n}}$$

$$= 5 \cdot e^{\lim_{n \rightarrow +\infty} \left(\frac{4}{5} \right)^n \cdot \frac{1}{n}} = 5 \cdot e^{0.0} = 5$$

اذن: $\lim_{n \rightarrow +\infty} \sqrt[n]{4^n + 5^n} = 5$

نهاية 7: $\lim_{x \rightarrow 1} \frac{\sin(\pi x) + \cos\left(\frac{\pi x}{2}\right)}{(1+3x) \cdot \cos\left(\frac{3\pi x}{2}\right)}$

نعلم أن: $\lim_{x \rightarrow a} \frac{\sin(bx)}{\sin(dx)} = \frac{b}{d}$

$$\lim_{x \rightarrow 1} \frac{\sin(\pi x) + \cos\left(\frac{\pi x}{2}\right)}{(1+3x) \cdot \cos\left(\frac{3\pi x}{2}\right)} = \lim_{x \rightarrow 1} \frac{\sin(\pi x) + \sin\left(\frac{\pi}{2} - \frac{\pi x}{2}\right)}{(1+3x) \cdot \sin\left(\frac{\pi}{2} - \frac{\pi x}{2}\right)}$$

$$= \lim_{x \rightarrow 1} \frac{\sin(\pi x)}{(1+3x) \cdot \sin \frac{\pi}{2}(1-x)} + \lim_{x \rightarrow 1} \frac{\sin \frac{\pi}{2}(1-x)}{(1+3x) \cdot \sin \frac{\pi}{2}(1-x)}$$

$$= e^{\lim_{x \rightarrow 0} \frac{\tan\left(\frac{\pi}{4} + x - \frac{\pi}{4}\right) \left(1 + \tan\left(\frac{\pi}{4} + x\right)\right) \cdot \tan\left(\frac{\pi}{4}\right)}{x}}$$

$$= e^{\lim_{x \rightarrow 0} \frac{\tan x}{x} \cdot \left(1 + \tan\left(\frac{\pi}{4} + x\right)\right)}$$

$$= e^2$$

و بالتالي: $\lim_{x \rightarrow 0} \tan\left(\frac{\pi}{4} + x\right)^{\frac{1}{x}} = e^2$

نهاية 12 : $\lim_{x \rightarrow +\infty} \left(\cos\left(\frac{1}{x}\right) + \sin\left(\frac{1}{x^2}\right) \right)^{x^2}$

نعلم أن: $\lim_{x \rightarrow 0} \frac{\ln(1+x)}{x} = 1$

نضع: $x = \frac{1}{y}, x \rightarrow +\infty, y \rightarrow 0$

$$\lim_{x \rightarrow +\infty} \left(\cos\left(\frac{1}{x}\right) + \sin\left(\frac{1}{x^2}\right) \right)^{x^2}$$

$$= \lim_{y \rightarrow 0} \left(\cos(y) + \sin(y^2) \right)^{\frac{1}{y^2}}$$

$$= e^{\lim_{y \rightarrow 0} \frac{1}{y^2} \ln(\cos(y) + \sin(y^2))}$$

$$= e^{\lim_{y \rightarrow 0} \frac{\ln(\cos(y) + \sin(y^2))}{y^2}}$$

$$= e^{\lim_{y \rightarrow 0} \frac{\ln(\cos(y) + \sin(y^2))}{\cos(y) + \sin(y^2) - 1} \cdot \frac{\cos(y) + \sin(y^2) - 1}{y^2}}$$

$$= e^{\lim_{y \rightarrow 0} \frac{(\cos(y) + \sin(y^2) - 1)}{y^2}}$$

$$= e^{\lim_{y \rightarrow 0} \frac{-2\sin\left(\frac{y^2}{2}\right) + \sin(y^2)}{y^2}}$$

$$= e^{-2 \cdot \frac{1}{4} + 1} = e^{\frac{1}{2}} = \sqrt{e}$$

و بالتالي: $\lim_{x \rightarrow +\infty} \left(\cos\left(\frac{1}{x}\right) + \sin\left(\frac{1}{x^2}\right) \right)^{x^2} = \sqrt{e}$

نهاية 10: $\lim_{x \rightarrow 0} \frac{(2-x^2)\sin(x) - \sin(2x)}{x^5}$

نعلم أن: $\lim_{x \rightarrow 0} \frac{\sin(x) - x}{x^3} = \frac{-1}{6}$

$$\lim_{x \rightarrow 0} \frac{(2-x^2)\sin(x) - \sin(2x)}{x^5}$$

$$= \lim_{x \rightarrow 0} \frac{2\sin(x) - x^2\sin(x) - 2\sin(x)\cos(x)}{x^5}$$

$$= \lim_{x \rightarrow 0} \frac{2\sin(x)(1 - \cos(x) - x^2\sin(x))}{x^5}$$

$$= \lim_{x \rightarrow 0} \frac{\sin x}{x} \cdot \frac{2(2)\sin^2\left(\frac{x}{2}\right) - x^2}{x^4}$$

$$= 1 \cdot \lim_{x \rightarrow 0} \frac{4\sin^2\left(\frac{x}{2}\right) - x^2}{x^4}$$

$$= \lim_{x \rightarrow 0} \frac{\left(2\sin^2\left(\frac{x}{2}\right) + x\right) \left(2\sin^2\left(\frac{x}{2}\right) - x\right)}{x \cdot x^3}$$

$$= \lim_{x \rightarrow 0} \left(\frac{2\sin^2\left(\frac{x}{2}\right)}{x} + \frac{x}{x} \right) \cdot \frac{\left(2\sin^2\left(\frac{x}{2}\right) - \frac{2x}{2}\right)}{x^3}$$

$$= \left(2 \cdot \left(\frac{1}{2}\right) + 1 \right) \cdot 2 \lim_{x \rightarrow 0} \frac{\sin\left(\frac{x}{2}\right) - \left(\frac{x}{2}\right) \cdot \left(\frac{x}{2}\right)^3}{\left(\frac{x}{2}\right)^3 \cdot x^3}$$

$$= 2 \cdot 2 \cdot \frac{-1}{6} \cdot \frac{1}{8} = \frac{-1}{12}$$

و بالتالي: $\lim_{x \rightarrow 0} \frac{(2-x^2)\sin(x) - \sin(2x)}{x^5} = -\frac{1}{12}$

نهاية 11: $\lim_{x \rightarrow 0} \tan\left(\frac{\pi}{4} + x\right)^{\frac{1}{x}}$

$$\lim_{x \rightarrow 0} \tan\left(\frac{\pi}{4} + x\right)^{\frac{1}{x}} = \lim_{x \rightarrow 0} \left(1 + \tan\left(\frac{\pi}{4} + x\right) - 1 \right)^{\frac{1}{x}}$$

$$= e^{\lim_{x \rightarrow 0} \frac{\tan\left(\frac{\pi}{4} + x\right) - 1}{x}} = e^{\lim_{x \rightarrow 0} \frac{\tan\left(\frac{\pi}{4} + x\right) - \tan\left(\frac{\pi}{4}\right)}{x}}$$

$$= \lim_{n \rightarrow +\infty} \frac{1}{n^2} \left(\frac{n(n-1)}{2} \right) \text{ و بالتالي:}$$

$$= \lim_{n \rightarrow +\infty} \frac{n^2 - n}{2n^2} = \frac{1}{2}$$

نهاية 15 : $\lim_{x \rightarrow \frac{\pi}{2}} (\sin x)^{\frac{1}{\cos x}}$

نعلم أن: $\lim_{x \rightarrow 1} \frac{\ln x}{x-1} = 1$

$$\lim_{x \rightarrow \frac{\pi}{2}} (\sin x)^{\frac{1}{\cos x}} = e^{\lim_{x \rightarrow \frac{\pi}{2}} \frac{1}{\cos x} \ln(\sin x)}$$

$$= e^{\lim_{x \rightarrow \frac{\pi}{2}} \frac{\ln(\sin x)}{\cos x}} = e^{\lim_{x \rightarrow \frac{\pi}{2}} \frac{\ln(\sin x)}{\sin x - 1} \times \frac{\sin x - 1}{\cos x}}$$

$$= e^{1 \times \lim_{x \rightarrow \frac{\pi}{2}} \frac{(\sin x - 1)}{(\sin x + 1)} \times \frac{(\sin x + 1)}{\cos x}}$$

$$= e^{\lim_{x \rightarrow \frac{\pi}{2}} \frac{\cos^2 x}{\cos x (\sin x + 1)}}$$

$$= e^{\frac{0}{2}} = 1$$

اذن: $\lim_{x \rightarrow \frac{\pi}{2}} (\sin x)^{\frac{1}{\cos x}} = 1$

نهاية 16 : $\lim_{x \rightarrow 0} \frac{1 - \cos x \sqrt{\cos 2x}}{x^2}$

$$\lim_{x \rightarrow 0} \frac{1 - \cos x \sqrt{\cos 2x}}{x^2} = \lim_{x \rightarrow 0} \frac{1 - \sqrt{\cos^2 x \cos 2x}}{x^2}$$

$$\lim_{x \rightarrow 0} \frac{1 - \sqrt{\cos^2 x \cos 2x}}{1 - (\cos^2 x \cos x)} \times \frac{1 - (\cos^2 x \cos x)}{x^2}$$

$$= \frac{1}{2} (1)^{\frac{1}{2}-1} \times \lim_{x \rightarrow 0} \frac{1 - (1 - \sin^2 x) \cos 2x}{x^2}$$

نهاية 13 : $\lim_{x \rightarrow 2^-} \frac{x - 2 \cos(\sqrt{2-x})}{(2-x)^2}$

نعلم أن: $\lim_{x \rightarrow 0} \frac{\sin(x) - x}{x^3} = \frac{-1}{6}$

نضع: $y = 2 - x, x \rightarrow 2^-, y \rightarrow 0$

$$\lim_{x \rightarrow 2^-} \frac{x - 2 \cos(\sqrt{2-x})}{(2-x)^2}$$

$$= \lim_{y \rightarrow 0} \frac{2 - y - 2 \cos(\sqrt{y})}{y^2}$$

$$= \lim_{y \rightarrow 0} \frac{2(1 - \cos(\sqrt{y})) - y}{y^2}$$

$$= \lim_{y \rightarrow 0} \frac{2 \left(2 \sin^2 \left(\frac{\sqrt{y}}{2} \right) \right) - y}{y^2}$$

$$= \lim_{y \rightarrow 0} \frac{1}{16} \frac{4 \sin^2 \left(\frac{\sqrt{y}}{2} \right) - \left(\frac{\sqrt{y}}{2} \right)^2}{y^4}$$

$$= \frac{4}{16} \lim_{y \rightarrow 0} \frac{\sin \left(\frac{\sqrt{y}}{2} \right) + \left(\frac{\sqrt{y}}{2} \right)}{y} \times \frac{\sin \left(\frac{\sqrt{y}}{2} \right) - \left(\frac{\sqrt{y}}{2} \right)}{y^3}$$

$$= \frac{1}{4} \cdot 2 \cdot \frac{-1}{6} = \frac{-1}{12}$$

و منه: $\lim_{x \rightarrow 2^-} \frac{x - 2 \cos(\sqrt{2-x})}{(2-x)^2} = \frac{-1}{12}$

نهاية 14 : $\lim_{n \rightarrow +\infty} \frac{1}{n^2} + \frac{2}{n^2} + \dots + \frac{n-1}{n^2}$

نعلم أن: $1 + 2 + 3 + \dots + (n-1) = \frac{n(n-1)}{2}$

$$\lim_{n \rightarrow +\infty} \frac{1}{n^2} + \frac{2}{n^2} + \dots + \frac{n-1}{n^2}$$

$$= \lim_{n \rightarrow +\infty} \frac{1}{n^2} (1 + 2 + 3 + \dots + (n-1))$$

نعلم أن: $\lim_{x \rightarrow 0} \frac{\ln(1+x)}{x} = 1$

$$= \lim_{x \rightarrow 0} \ln(x) \ln\left(1 + \frac{x}{\ln x}\right)$$

$$= \lim_{x \rightarrow 0} \ln(x) \cdot \frac{\ln\left(1 + \frac{x}{\ln x}\right)}{\frac{x}{\ln x}} \cdot \frac{x}{\ln x}$$

و بالتالي:

$$= \lim_{x \rightarrow 0} \ln(x) \cdot (1) \cdot \frac{x}{\ln x}$$

$$= 0$$

و منه: $\lim_{x \rightarrow 0} x^{\ln\left(1 + \frac{x}{\ln x}\right)} = e^0 = 1$

اذن: $\lim_{x \rightarrow 0} x^{\ln\left(1 + \frac{x}{\ln x}\right)} = 1$

نهاية 20: $\lim_{n \rightarrow +\infty} e^{-x} \sin(e^x)$

$$\lim_{n \rightarrow +\infty} e^{-x} \sin(e^x) = \lim_{n \rightarrow +\infty} \frac{\sin(e^x)}{e^x}$$

$$\lim_{n \rightarrow +\infty} \frac{1}{e^x} \rightarrow 0$$

و لدينا: $|\sin(e^x)| \leq 1$

و نعلم أن نهاية دالة صفرية في دالة محدودة تساوي 0

اذن: $\lim_{n \rightarrow +\infty} e^{-x} \sin(e^x) = 0$

نهاية 21: $\lim_{x \rightarrow 0} \frac{1 - \cos(\sin x)}{x^2}$

$$\lim_{x \rightarrow 0} \frac{1 - \cos(\sin x)}{x^2}$$

$$= \lim_{x \rightarrow 0} \frac{1 - \cos(\sin x)}{x^2} \cdot \frac{1 + \cos(\sin x)}{1 + \cos(\sin x)}$$

$$= \lim_{x \rightarrow 0} \frac{1 - \cos^2(\sin x)}{x^2} \cdot \frac{1}{1 + \cos(\sin x)}$$

$$= \frac{1}{2} \lim_{x \rightarrow 0} \frac{1 - \cos 2x + \cos 2x (\sin^2 x)}{x^2}$$

$$= \frac{1}{2} \lim_{x \rightarrow 0} \left(\frac{2(\sin^2 x)}{x^2} + \frac{(\sin^2 x)}{x^2} \cos 2x \right)$$

$$= \frac{1}{2} (2 + 1 \times 1) = \frac{3}{2}$$

و بالتالي: $\lim_{x \rightarrow 0} \frac{1 - \cos x \sqrt{\cos 2x}}{x^2} = \frac{3}{2}$

نهاية 17: $\lim_{x \rightarrow +\infty} \frac{e^x}{\ln x}$

$$\lim_{x \rightarrow +\infty} \frac{e^x}{\ln x} = \lim_{x \rightarrow +\infty} \left(\frac{e^x}{x} \right) \times \left(\frac{x}{\ln x} \right) = +\infty$$

نهاية 18: $\lim_{x \rightarrow +\infty} \left(\frac{x+2}{x} \right)^x$

$$\lim_{x \rightarrow +\infty} \left(\frac{x+2}{x} \right)^x = e^{\lim_{x \rightarrow +\infty} x \ln \left(\frac{x+2}{x} \right)}$$

$$= e^{\lim_{x \rightarrow +\infty} x \ln \left(1 + \frac{2}{x} \right)} = e^{\lim_{x \rightarrow +\infty} 2 \frac{\ln \left(1 + \frac{2}{x} \right)}{\frac{x}{2}}}$$

نضع: $t = \frac{2}{x}, x \rightarrow +\infty, t \rightarrow 0$

$$\lim_{x \rightarrow +\infty} 2 \frac{\ln \left(1 + \frac{2}{x} \right)}{\frac{x}{2}} = \lim_{x \rightarrow +\infty} 2 \frac{\ln(1+t)}{t} = e^2$$

و بالتالي: $\lim_{x \rightarrow +\infty} \left(\frac{x+2}{x} \right)^x = e^2$

نهاية 19: $\lim_{x \rightarrow 0} x^{\ln \left(1 + \frac{x}{\ln x} \right)}$

$$\lim_{x \rightarrow 0} x^{\ln \left(1 + \frac{x}{\ln x} \right)} = \lim_{x \rightarrow 0} e^{\ln \left(x^{\ln \left(1 + \frac{x}{\ln x} \right)} \right)}$$

$$= \lim_{x \rightarrow 0} e^{\ln(x^1) \ln \left(1 + \frac{x}{\ln x} \right)}$$

$$\begin{aligned}\lim_{x \rightarrow \infty} \frac{\tan\left(\frac{1}{x} + \frac{\pi}{2}\right)}{x} &= \lim_{x \rightarrow \infty} \frac{\cot\left(-\frac{1}{x}\right)}{x} \\ &= -1 \times \lim_{x \rightarrow \infty} \frac{\cot\left(\frac{1}{x}\right)}{x} \\ &= -1 \times \lim_{x \rightarrow \infty} \frac{1}{x \tan\left(\frac{1}{x}\right)} \\ &= -1 \times \lim_{x \rightarrow \infty} \frac{\frac{1}{x}}{\tan\left(\frac{1}{x}\right)} = -1\end{aligned}$$

$$\lim_{x \rightarrow \infty} \frac{\tan\left(\frac{1}{x} + \frac{\pi}{2}\right)}{x} = -1 \quad \text{اذن :}$$

$$\lim_{x \rightarrow \frac{\pi}{2}} \frac{1 - \sin x}{\frac{\pi}{2} - x} \quad \text{نهاية 24 :}$$

نعلم أن:

$$1 - \sin x = \sin \frac{\pi}{2} - \sin x = 2 \cos\left(\frac{\pi}{4} + \frac{x}{2}\right) \cdot \sin\left(\frac{\pi}{4} - \frac{x}{2}\right)$$

$$\lim_{x \rightarrow \frac{\pi}{2}} \frac{1 - \sin x}{\frac{\pi}{2} - x} = \lim_{x \rightarrow \frac{\pi}{2}} \frac{2 \cos\left(\frac{\pi}{4} + \frac{x}{2}\right) \cdot \sin\left(\frac{\pi}{4} - \frac{x}{2}\right)}{\frac{\pi}{2} - x}$$

$$= 4 \cos\left(\frac{\pi}{2}\right) \cdot \lim_{x \rightarrow \frac{\pi}{2}} \frac{\sin \frac{1}{4}(\pi - 2x)}{\frac{1}{4}(\pi - 2x)} \cdot \frac{1}{4}$$

$$= 0$$

$$\lim_{x \rightarrow \frac{\pi}{2}} \frac{1 - \sin x}{\frac{\pi}{2} - x} = 0 \quad \text{اذن :}$$

$$\lim_{x \rightarrow 0} \frac{x - \sin^{-1} x}{x - \sin x} \quad \text{نهاية 25 :}$$

$$\begin{aligned}x &= \sin y \\ y &= \sin^{-1} x\end{aligned} \quad \text{نضع :}$$

$$\begin{aligned}&= \lim_{x \rightarrow 0} \frac{\sin^2(\sin x)}{x^2} \cdot \frac{\sin^2(x)}{\sin^2(x)} \cdot \frac{1}{2} \\ &= \lim_{x \rightarrow 0} \frac{\sin^2(\sin x)}{\sin^2(x)} \cdot \frac{\sin^2(x)}{x^2} \cdot \frac{1}{2} \\ &= \frac{1}{2}\end{aligned}$$

$$\lim_{x \rightarrow 0} \frac{1 - \cos(\sin x)}{x^2} = \frac{1}{2} \quad \text{اذن :}$$

$$\lim_{x \rightarrow 1} \frac{1}{\ln x} - \frac{1}{1-x} \quad \text{نهاية 22 :}$$

$$\begin{aligned}\lim_{x \rightarrow 1} \frac{e^x - x - 1}{x^2} &= \frac{1}{2} \\ \lim_{x \rightarrow 1} \frac{e^x - 1}{x} &= 1\end{aligned} \quad \text{نعلم أن :}$$

$$x = e^y \quad \text{نضع :}$$

$$\begin{aligned}\lim_{x \rightarrow 1} \frac{1}{\ln x} - \frac{1}{1-x} &= \lim_{x \rightarrow 1} \frac{1-x-\ln x}{(1-x)\ln x} \\ &= \lim_{y \rightarrow 0} \left(\frac{-(e^y - y - 1)}{(1-e^y) \cdot y} \right) \\ &= \lim_{y \rightarrow 0} \frac{-(e^y - y - 1)}{y^2} \cdot \frac{y^2}{(1-e^y) \cdot y} \\ &= \frac{-1}{2} \times -1 \\ &= \frac{1}{2}\end{aligned}$$

$$\lim_{x \rightarrow 1} \frac{1}{\ln x} - \frac{1}{1-x} = \frac{1}{2} \quad \text{و بالتالي :}$$

$$\lim_{m \rightarrow \infty} \frac{\tan\left(\frac{1}{x} + \frac{\pi}{2}\right)}{x} \quad \text{نهاية 23 :}$$

$$\begin{aligned}
&= \lim_{x \rightarrow 0^+} \frac{\sin(\sqrt{\sin \pi x})}{\sqrt{\pi x}} \times \lim_{x \rightarrow 0^+} \frac{\sqrt{\pi x}}{\sqrt{x}(\sqrt{x}-1)} \\
&= 1 \cdot \lim_{x \rightarrow 0^+} \frac{\sqrt{\pi} \cdot \sqrt{x}}{\sqrt{x}(\sqrt{x}-1)} \\
&= \lim_{x \rightarrow 0^+} \frac{\sqrt{\pi}}{(\sqrt{x}-1)} \\
&= -\sqrt{\pi} \\
&\quad \cdot \lim_{x \rightarrow 0^+} \frac{\sin(\sqrt{\sin \pi x})}{x - \sqrt{x}} = -\sqrt{\pi} \quad \text{اذن:}
\end{aligned}$$

نهاية 27: $a > 0, \lim_{x \rightarrow a} \frac{\sqrt{x} - \sqrt{a} + \sqrt{x-a}}{\sqrt{x^2 - a^2}}$

$$\begin{aligned}
\lim_{x \rightarrow a} \frac{\sqrt{x} - \sqrt{a} + \sqrt{x-a}}{\sqrt{x^2 - a^2}} &= \lim_{x \rightarrow a} \frac{\sqrt{x} - \sqrt{a} + \sqrt{x-a}}{(x+a)(x-a)} \\
&= \lim_{x \rightarrow a} \frac{\sqrt{x-a} \left(\frac{\sqrt{x} - \sqrt{a}}{\sqrt{x-a}} + 1 \right)}{(\sqrt{x+a})(\sqrt{x-a})} \\
&= \lim_{x \rightarrow a} \frac{\left(\frac{\sqrt{x} - \sqrt{a}}{\sqrt{x-a}} + 1 \right)}{(\sqrt{x+a})} \\
&= \lim_{x \rightarrow a} \frac{1}{(\sqrt{x+a})} \times \lim_{x \rightarrow a} \left(\frac{\sqrt{x} - \sqrt{a}}{\sqrt{x-a}} + 1 \right) \\
&= \frac{1}{\sqrt{2a}} \times \lim_{x \rightarrow a} \left(\frac{\sqrt{x} - \sqrt{a}}{\sqrt{x-a}} \times \frac{\sqrt{x-a}}{\sqrt{x-a}} + 1 \right) \\
&= \frac{1}{\sqrt{2a}} \times \lim_{x \rightarrow a} \left(\frac{(\sqrt{x} - \sqrt{a})\sqrt{x-a}}{x-a} + 1 \right) \\
&= \frac{1}{\sqrt{2a}} \times \lim_{x \rightarrow a} \left(\frac{(\sqrt{x} - \sqrt{a})\sqrt{x-a}}{(\sqrt{x} - \sqrt{a})(\sqrt{x} + \sqrt{a})} + 1 \right) \\
&= \frac{1}{\sqrt{2a}} \times \lim_{x \rightarrow a} \left(\frac{\sqrt{x-a}}{(\sqrt{x} + \sqrt{a})} + 1 \right)
\end{aligned}$$

$$\lim_{x \rightarrow 0} \frac{x - \sin^{-1} x}{x - \sin x} = \lim_{y \rightarrow 0} \frac{\sin y - y}{\sin y - \sin(\sin y)}$$

نعلم أن:

$$\sin y - \sin(\sin y) = 2 \cos\left(\frac{y + \sin y}{2}\right) \sin\left(\frac{y - \sin y}{2}\right)$$

$$\begin{aligned}
&\lim_{y \rightarrow 0} \frac{\sin y - y}{\sin y - \sin(\sin y)} \\
&= \lim_{y \rightarrow 0} \frac{\sin y - y}{2 \cos\left(\frac{y + \sin y}{2}\right) \sin\left(\frac{y - \sin y}{2}\right)}
\end{aligned}$$

$$\begin{aligned}
&\lim_{y \rightarrow 0} \frac{\frac{\sin y - y}{2}}{\cos\left(\frac{y + \sin y}{2}\right) \sin\left(\frac{y - \sin y}{2}\right)} \\
&= \lim_{y \rightarrow 0} \frac{\frac{\sin y - y}{2}}{\sin\left(\frac{y - \sin y}{2}\right)} \times \lim_{y \rightarrow 0} \frac{1}{\cos\left(\frac{y + \sin y}{2}\right)} \\
&= -1 \lim_{y \rightarrow 0} \frac{2}{\sin\left(\frac{y - \sin y}{2}\right)} \times \lim_{y \rightarrow 0} \frac{1}{\cos\left(\frac{y + \sin y}{2}\right)} \\
&= -1.1.1 = -1
\end{aligned}$$

اذن: $\lim_{x \rightarrow 0} \frac{x - \sin^{-1} x}{x - \sin x} = -1$

نهاية 26: $\lim_{x \rightarrow 0^+} \frac{\sin(\sqrt{\sin \pi x})}{x - \sqrt{x}}$

$$\begin{aligned}
\lim_{x \rightarrow 0^+} \frac{\sin(\sqrt{\sin \pi x})}{x - \sqrt{x}} &= \lim_{x \rightarrow 0^+} \frac{\sin(\sqrt{\sin \pi x})}{x - \sqrt{x}} \cdot \frac{\sqrt{\sin \pi x}}{\sqrt{\sin \pi x}} \\
&= \lim_{x \rightarrow 0^+} \frac{\sin(\sqrt{\sin \pi x})}{\sqrt{\sin \pi x}} \times \lim_{x \rightarrow 0^+} \frac{\sqrt{\sin \pi x}}{x - \sqrt{x}} \times \frac{\sqrt{\pi x}}{\sqrt{\pi x}} \\
&= 1 \times \lim_{x \rightarrow 0^+} \frac{\sin(\sqrt{\sin \pi x})}{\sqrt{\pi x}} \times \lim_{x \rightarrow 0^+} \frac{\sqrt{\pi x}}{x - \sqrt{x}}
\end{aligned}$$

$$\lim_{t \rightarrow 0} \frac{\ln(1+\sqrt{t})}{t} - \frac{1}{\sqrt{t}}$$

$$= \lim_{t \rightarrow 0} \frac{\ln(1+\sqrt{t}) - \sqrt{t} - 0}{t - 0}$$

نهاية العدد المشتق حيث: $f(t) = \ln(1+\sqrt{t}) - \sqrt{t}$

$$\lim_{t \rightarrow 0} \frac{\ln(1+\sqrt{t}) - \sqrt{t} - 0}{t - 0} = f'(0) = \frac{-1}{2}$$

$$\lim_{x \rightarrow +\infty} x^2 \ln\left(\frac{x+1}{x}\right) - x = \frac{-1}{2} \text{ اذن:}$$

$$\lim_{x \rightarrow \frac{\pi}{4}} \frac{4\sqrt{2} - (\cos x + \sin x)^5}{1 - 2 \sin x} \text{ نهاية 30:}$$

$$\lim_{x \rightarrow \frac{\pi}{4}} \frac{4\sqrt{2} - (\cos x + \sin x)^5}{1 - 2 \sin x}$$

$$= \lim_{x \rightarrow \frac{\pi}{4}} \frac{4\sqrt{2} - 4\sqrt{2} \left(\frac{1}{\sqrt{2}} \cos x + \frac{1}{\sqrt{2}} \sin x \right)^5}{1 - \cos\left(\frac{\pi}{2} - 2x\right)}$$

$$= \lim_{x \rightarrow \frac{\pi}{4}} \frac{4\sqrt{2} \left(1 - \cos^5\left(\frac{\pi}{4} - x\right) \right)}{1 - 2 \cos^2\left(\frac{\frac{\pi}{2} - 2x}{2}\right) - 1}$$

$$= \lim_{x \rightarrow \frac{\pi}{4}} \frac{4\sqrt{2} \left(1 - \cos^5\left(\frac{\pi}{4} - x\right) \right)}{2 \cos^2\left(\frac{\frac{\pi}{2} - 2x}{2}\right)}$$

$$= \frac{4\sqrt{2}}{2} \cdot \frac{5}{2} \cdot (1)^{5-2} = 5\sqrt{2}.$$

$$= \frac{1}{\sqrt{2a}} \times (0+1)$$

$$= \frac{1}{\sqrt{2a}}$$

$$\text{اذن: } a > 0, \lim_{x \rightarrow a} \frac{\sqrt{x} - \sqrt{a} + \sqrt{x-a}}{\sqrt{x^2 - a^2}} = \frac{1}{\sqrt{2a}}$$

$$\lim_{x \rightarrow \pi} \frac{1 - \sin \frac{x}{2}}{x - \pi} \text{ نهاية 28:}$$

$$1 + \sin x = 2 \cos^2\left(\frac{\pi}{4} - \frac{x}{2}\right)$$

$$1 - \sin x = 2 \sin^2\left(\frac{\pi}{4} - \frac{x}{2}\right)$$

نعلم أن:

$$\lim_{x \rightarrow \pi} \frac{1 - \sin \frac{x}{2}}{x - \pi} = \lim_{x \rightarrow \pi} \frac{2 \sin^2\left(\frac{\pi}{4} - \frac{x}{4}\right)}{4\left(\frac{x}{4} - \frac{\pi}{4}\right)}$$

$$= \lim_{x \rightarrow \pi} \frac{2 \sin\left(\frac{\pi}{4} - \frac{x}{4}\right)}{-4\left(\frac{\pi}{4} - \frac{x}{4}\right)} \times \sin\left(\frac{\pi}{4} - \frac{x}{4}\right)$$

و بالتالي:

$$= \frac{-1}{2} \cdot 0 = 0$$

$$\lim_{x \rightarrow \pi} \frac{1 - \sin \frac{x}{2}}{x - \pi} = 0 \text{ اذن:}$$

$$\lim_{x \rightarrow +\infty} x^2 \ln\left(\frac{x+1}{x}\right) - x \text{ نهاية 29:}$$

$$\lim_{x \rightarrow +\infty} x^2 \ln\left(\frac{x+1}{x}\right) - x = \lim_{x \rightarrow +\infty} \frac{\ln\left(\frac{x+1}{x}\right)}{\frac{1}{x^2}} - x$$

$$\text{نضع: } t = \frac{1}{x^2} \text{ اذن: } t \rightarrow 0$$

$$= \lim_{x \rightarrow 0} \frac{e^{\ln 2 \cdot \cos x} \left(e^{\ln 2 \left(\frac{1 - \cos^2 x}{\cos x} \right)} - 1 \right)}{\ln 2 \left(\frac{\sin^2 x}{\cos x} \right)} \times \frac{\ln 2 \left(\frac{\sin^2 x}{x^2} \right)}{\cos x}$$

$$= 2 \ln 2$$

اذن: $\lim_{x \rightarrow 0} \frac{2^{\sec x} - 2^{\cos x}}{x^2} = 2 \ln 2$

نهاية 33: $\lim_{x \rightarrow 0} \frac{1}{x \sin x} - \frac{1}{x^2}$

نعم أن: $\lim_{x \rightarrow 0} \frac{x - \sin x}{x^3} = \frac{1}{6}$

$$\lim_{x \rightarrow 0} \frac{1}{x \sin x} - \frac{1}{x^2} = \lim_{x \rightarrow 0} \frac{x - \sin x}{x^2 \sin x}$$

$$= \lim_{x \rightarrow 0} \frac{x - \sin x}{x^3} \times \frac{x^3}{x^2 \sin x}$$

$$= \lim_{x \rightarrow 0} \frac{x - \sin x}{x^3} \times \frac{x}{\sin x}$$

$$= \frac{1}{6} \cdot 1 = \frac{1}{6}$$

و بالتالي: $\lim_{x \rightarrow 0} \frac{1}{x \sin x} - \frac{1}{x^2} = \frac{1}{6}$

نهاية 34: $\lim_{x \rightarrow e} \frac{x - e \ln x}{(x - e)^2}$

ليكن: $\lim_{x \rightarrow e} \frac{x - \ln x - 1}{(x - 1)^2} = \frac{1}{2}$

بوضع: $x = ey$
 $x \rightarrow e, y \rightarrow 1$

$$\lim_{x \rightarrow e} \frac{x - e \ln x}{(x - e)^2} = \lim_{y \rightarrow 1} \frac{ey - e \ln ey}{(ey - e)^2}$$

$$= \lim_{y \rightarrow 1} \frac{e}{e^2} \times \frac{(y - \ln e - \ln y)}{(y - 1)^2}$$

اذن: $\lim_{x \rightarrow \frac{\pi}{4}} \frac{4\sqrt{2} - (\cos x + \sin x)^5}{1 - 2 \sin x} = 5\sqrt{2}$

نهاية 31: $\lim_{x \rightarrow \frac{\pi}{2}} (\sin x)^{\tan x}$

نضع: $y = \frac{\pi}{2} - x$

$$\lim_{y \rightarrow 0} \left(\sin \left(\frac{\pi}{2} - y \right) \right)^{\tan \left(\frac{\pi}{2} - y \right)}$$

$$= \lim_{y \rightarrow 0} (\cos y)^{\cot y}$$

$$= \lim_{y \rightarrow 0} (1 + (\cos y - 1))^{\cot y}$$

$$= \lim_{y \rightarrow 0} \left(1 - 2 \sin^2 \left(\frac{y}{2} \right) \right)^{\cot y}$$

$$= \lim_{y \rightarrow 0} \left(1 - 2 \sin^2 \left(\frac{y}{2} \right) \right)^{\frac{\cos y}{\sin y}}$$

$$= e^{\lim_{y \rightarrow 0} \ln \left(1 - 2 \sin^2 \left(\frac{y}{2} \right) \right)^{\frac{\cos y}{\sin y}}}$$

$$= e^{\lim_{y \rightarrow 0} \frac{\cos y}{\sin y} \times \ln \left(1 - 2 \sin^2 \left(\frac{y}{2} \right) \right)}$$

$$= e^0 = 1$$

$\lim_{x \rightarrow \frac{\pi}{2}} (\sin x)^{\tan x} = 1$

نهاية 32: $\lim_{x \rightarrow 0} \frac{2^{\sec x} - 2^{\cos x}}{x^2}$

نعم أن: $\lim_{x \rightarrow 0} \frac{e^x - 1}{x} = 1$

$$\lim_{x \rightarrow 0} \frac{2^{\sec x} - 2^{\cos x}}{x^2} = \lim_{x \rightarrow 0} \frac{e^{\ln 2 \cdot \sec x} - e^{\ln 2 \cdot \cos x}}{x^2}$$

$$= \lim_{x \rightarrow 0} \frac{e^{\ln 2 \cdot \cos x} \left(e^{\ln 2 \cdot \frac{1}{\cos x} \cdot \ln 2 \cdot \cos x} - 1 \right)}{x^2}$$

$$= \lim_{x \rightarrow 0} \frac{\sin^2\left(\frac{3x}{2}\right)}{\sin^2(2x)} = \frac{9}{16}$$

$$\cdot \lim_{x \rightarrow 0} \frac{1 - \cos(3x)}{1 - \cos(4x)} = \frac{9}{16} \text{ ومنه:}$$

نهاية 37: $\lim_{x \rightarrow 1} \frac{x\sqrt{x+3}-2}{x^2+x-2}$

$$\lim_{x \rightarrow 1} \frac{x\sqrt{x+3}-2}{x^2+x-2}$$

$$= \lim_{x \rightarrow 1} \frac{x\sqrt{x+3}-\sqrt{x+3}+\sqrt{x+3}-2}{(x-2)(x-1)}$$

$$= \lim_{x \rightarrow 1} \frac{(x-1)\sqrt{x+3}}{(x-2)(x-1)} + \lim_{x \rightarrow 1} \frac{(x+3)^{\frac{1}{2}}-4^{\frac{1}{2}}}{(x+3-4)} \cdot \frac{1}{x+2}$$

$$= \frac{2}{3} + \frac{1}{12} = \frac{3}{4}$$

$$\cdot \lim_{x \rightarrow 1} \frac{x\sqrt{x+3}-2}{x^2+x-2} = \frac{3}{4} \text{ إذن:}$$

نهاية 38: $\lim_{x \rightarrow \frac{\pi}{4}} \frac{\tan x - 1}{x - \frac{\pi}{4}}$

$$\cdot \tan a - \tan b = \frac{\sin(a-b)}{\cos a \cdot \cos b} \text{ نعلم أن:}$$

$$\lim_{x \rightarrow \frac{\pi}{4}} \frac{\tan x - 1}{x - \frac{\pi}{4}} = \lim_{x \rightarrow \frac{\pi}{4}} \frac{\tan x - \tan\left(\frac{\pi}{4}\right)}{\left(x - \frac{\pi}{4}\right)}$$

$$\lim_{x \rightarrow \frac{\pi}{4}} \frac{\sin\left(x - \frac{\pi}{4}\right)}{\left(x - \frac{\pi}{4}\right) \cos x \cdot \cos\left(\frac{\pi}{4}\right)}$$

$$= \frac{1}{e} \times \lim_{y \rightarrow 1} \frac{(y-1-\ln y)}{(y-1)^2}$$

$$= \frac{1}{e} \cdot \frac{1}{2}$$

$$= \frac{1}{2e}$$

$$\cdot \lim_{x \rightarrow e} \frac{x - e \ln x}{(x-e)^2} = \frac{1}{2e} \text{ ومنه:}$$

نهاية 35: $\lim_{x \rightarrow 0} \frac{x - \tan^{-1} x}{x^3}$

$$\cdot \lim_{x \rightarrow 0} \frac{x - \sin x}{x^3} = \frac{1}{6} \text{ نعلم أن:}$$

$$\cdot y = \tan^{-1} x \text{ نضع:}$$

$$\lim_{x \rightarrow 0} \frac{x - \tan^{-1} x}{x^3} = \lim_{y \rightarrow 0} \frac{\tan y - y}{\tan^3 y}$$

$$= \lim_{y \rightarrow 0} \frac{\sin y - y \cdot \cos y}{y^3 \cos y}$$

$$= \lim_{y \rightarrow 0} \frac{(\sin y - y) + (y - y \cdot \cos y)}{y^3} \times \frac{1}{\cos y}$$

$$= \lim_{y \rightarrow 0} \frac{(\sin y - y)}{y^3} \times \frac{1}{\cos y} + \lim_{y \rightarrow 0} \frac{\left(y \cdot 2 \sin^2\left(\frac{y}{2}\right)\right)}{y^3} \times \frac{1}{\cos y}$$

$$= \frac{-1}{6} \cdot 1 + 2 \cdot \frac{1}{4} = \frac{1}{3}$$

$$\cdot \lim_{x \rightarrow 0} \frac{x - \tan^{-1} x}{x^3} = \frac{1}{3} \text{ ومنه}$$

نهاية 36: $\lim_{x \rightarrow 0} \frac{1 - \cos(3x)}{1 - \cos(4x)}$

$$\lim_{x \rightarrow 0} \frac{1 - \cos(3x)}{1 - \cos(4x)} = \lim_{x \rightarrow 0} \frac{2 \sin^2\left(\frac{3x}{2}\right)}{2 \sin^2\left(\frac{4x}{2}\right)}$$

$$\lim_{x \rightarrow \frac{\pi}{6}} \frac{\sin(x) - \frac{1}{2}}{6x - \pi} = \lim_{x \rightarrow \frac{\pi}{6}} \frac{\sin(x) - \sin\left(\frac{\pi}{6}\right)}{6\left(x - \frac{\pi}{6}\right)}$$

$$= \lim_{x \rightarrow \frac{\pi}{6}} \frac{2 \sin\left(\frac{x - \frac{\pi}{6}}{2}\right) \cdot \cos\left(\frac{x + \frac{\pi}{6}}{2}\right)}{6\left(x - \frac{\pi}{6}\right)}$$

$$= \lim_{x \rightarrow \frac{\pi}{6}} \frac{2 \sin\left(\frac{x - \frac{\pi}{6}}{2}\right)}{\left(x - \frac{\pi}{6}\right)} \times \lim_{x \rightarrow \frac{\pi}{6}} \frac{\cos\left(\frac{x + \frac{\pi}{6}}{2}\right)}{6}$$

$$= 2 \cdot \frac{1}{2} \cdot \frac{\frac{\sqrt{3}}{2}}{6} = \frac{\sqrt{3}}{12}$$

$$\cdot \lim_{x \rightarrow \frac{\pi}{6}} \frac{\sin(x) - \frac{1}{2}}{6x - \pi} = \frac{\sqrt{3}}{12} \quad \text{و بالتالي:}$$

نهاية 41: $\lim_{x \rightarrow 0} \frac{\cos x - \sqrt[3]{\cos x}}{\sin^2 x}$

$$\lim_{x \rightarrow 0} \frac{\cos x - \sqrt[3]{\cos x}}{\sin^2 x} = \lim_{x \rightarrow 0} \frac{(\cos(x) - 1) - (\sqrt[3]{\cos x} - 1)}{\sin^2 x}$$

$$= -1 \times \lim_{x \rightarrow 0} \frac{(\cos(x) - 1) - (\sqrt[3]{\cos x} - 1)}{\cos^2(x) - 1}$$

$$= -1 \times \lim_{x \rightarrow 0} \frac{(\cos(x) - 1)}{\cos^2(x) - 1} + \lim_{x \rightarrow 0} \frac{(\sqrt[3]{\cos x} - 1)}{\cos^2(x) - 1}$$

$$= \frac{-1}{3}$$

$$\cdot \lim_{x \rightarrow 0} \frac{\cos x - \sqrt[3]{\cos x}}{\sin^2 x} = \frac{-1}{3} \quad \text{و بالتالي:}$$

$$\lim_{x \rightarrow \frac{\pi}{4}} \frac{\sin\left(x - \frac{\pi}{4}\right)}{\left(x - \frac{\pi}{4}\right)} \times \frac{1}{\cos x \cdot \cos\left(\frac{\pi}{4}\right)}$$

$$= 1 \cdot \frac{1}{\frac{\sqrt{2}}{2}} \cdot \frac{1}{\frac{\sqrt{2}}{2}} = 2$$

$$\cdot \lim_{x \rightarrow \frac{\pi}{4}} \frac{\tan x - 1}{\left(x - \frac{\pi}{4}\right)} = 2 \quad \text{و بالتالي:}$$

نهاية 39: $\lim_{x \rightarrow 0} \frac{\ln(2x^2 + \cos(2x))}{x^4}$

$$\lim_{x \rightarrow 0} \frac{\ln(2x^2 + \cos(2x))}{x^4}$$

$$= \lim_{x \rightarrow 0} \ln(2x^2 + \cos(2x))^{\frac{1}{x^4}}$$

$$= \lim_{x \rightarrow 0} \ln(2 + 2x^2 + \cos(2x) - 2)^{\frac{1}{x^4}}$$

$$= \ln e^{\lim_{x \rightarrow 0} \frac{(2x^2 - 2\sin^2 x)}{x^4}}$$

$$= \ln e^{\lim_{x \rightarrow 0} 2 \frac{(x - \sin x)}{x^4} \cdot \frac{(x + \sin x)}{x}}$$

$$= \frac{2}{3}$$

$$\cdot \lim_{x \rightarrow 0} \frac{\ln(2x^2 + \cos(2x))}{x^4} = \frac{2}{3} \quad \text{و منه:}$$

نهاية 40: $\lim_{x \rightarrow \frac{\pi}{6}} \frac{\sin(x) - \frac{1}{2}}{6x - \pi}$

و منه: $\lim_{x \rightarrow \sqrt{\pi}} \frac{1 + \cos \sqrt{\pi} x}{\pi - x^2} = 0$

نهاية 44: $\lim_{x \rightarrow \frac{\pi}{6}} \frac{1 - \sqrt{3} \tan x}{(\pi - 6x)}$

نعم أن: $\tan a - \tan b = \frac{\sin(a-b)}{\cos a \cdot \cos b}$

$$\begin{aligned} \lim_{x \rightarrow \frac{\pi}{6}} \frac{1 - \sqrt{3} \tan x}{(\pi - 6x)} &= \lim_{x \rightarrow \frac{\pi}{6}} \frac{\sqrt{3}}{6} \left(\frac{\frac{1}{\sqrt{3}} - \tan x}{\left(\frac{\pi}{6} - x\right)} \right) \\ &= \frac{\sqrt{3}}{6} \lim_{x \rightarrow \frac{\pi}{6}} \left(\frac{\frac{1}{\sqrt{3}} - \tan x}{\left(\frac{\pi}{6} - x\right)} \right) \\ &= \frac{\sqrt{3}}{6} \lim_{x \rightarrow \frac{\pi}{6}} \left(\frac{\tan\left(\frac{\pi}{6}\right) - \tan x}{\left(\frac{\pi}{6} - x\right)} \right) \\ &= \frac{\sqrt{3}}{6} \lim_{x \rightarrow \frac{\pi}{6}} \left(\frac{\frac{\sin\left(\frac{\pi}{6} - x\right)}{\cos\frac{\pi}{6} \cdot \cos x}}{\left(\frac{\pi}{6} - x\right)} \right) \\ &= \frac{2\sqrt{3}}{9} \end{aligned}$$

اذن: $\lim_{x \rightarrow \frac{\pi}{6}} \frac{1 - \sqrt{3} \tan x}{(\pi - 6x)} = \frac{2\sqrt{3}}{9}$

نهاية 45: $\lim_{x \rightarrow \infty} \left(\frac{x^2 + 5x + 4}{x^2 - 3x + 7} \right)^x$

نهاية 42: $\lim_{x \rightarrow 0} \frac{\ln\left(\frac{3 - \cos 2x - 2x^2}{2}\right)}{x^4}$

$$\begin{aligned} &\lim_{x \rightarrow 0} \frac{\ln\left(\frac{3 - \cos 2x - 2x^2}{2}\right)}{x^4} \\ &= \lim_{x \rightarrow 0} \ln\left(\frac{3 - \cos 2x - 2x^2}{2}\right)^{\frac{1}{x^4}} \\ &= \lim_{x \rightarrow 0} \ln\left(\frac{2}{2} + \frac{1 - \cos 2x - 2x^2}{2}\right)^{\frac{1}{x^4}} \\ &= \lim_{x \rightarrow 0} \ln\left(1 + \sin^2 x - x^2\right)^{\frac{1}{x^4}} \\ &= \ln \lim_{x \rightarrow 0} e^{\left(\frac{\sin x - x}{x^3} \cdot \frac{\sin x + x}{x}\right)} \\ &= \ln e^{-\frac{2}{6}} = \frac{-1}{3} \end{aligned}$$

اذن: $\lim_{x \rightarrow 0} \frac{\ln\left(\frac{3 - \cos 2x - 2x^2}{2}\right)}{x^4} = \frac{-1}{3}$

نهاية 43: $\lim_{x \rightarrow \sqrt{\pi}} \frac{1 + \cos \sqrt{\pi} x}{\pi - x^2}$

$$\begin{aligned} \lim_{x \rightarrow \sqrt{\pi}} \frac{1 + \cos \sqrt{\pi} x}{\pi - x^2} &= \lim_{x \rightarrow \sqrt{\pi}} \frac{2 \cos^2\left(\frac{\sqrt{\pi} x}{2}\right)}{\pi - x^2} \\ &= \lim_{x \rightarrow \sqrt{\pi}} \frac{2 \sin^2\left(\frac{\pi}{2} - \frac{\sqrt{\pi} x}{2}\right)}{\pi - x^2} \\ &= \lim_{x \rightarrow \sqrt{\pi}} 2 \left(\frac{\sin\left(\frac{\sqrt{\pi}(\sqrt{\pi} - x)}{2}\right)}{\sqrt{\pi} - x} \right) \times \left(\frac{\sin\left(\frac{\sqrt{\pi}(\sqrt{\pi} - x)}{2}\right)}{\sqrt{\pi} + x} \right) \\ &= 2 \times \frac{\sqrt{\pi}}{2} \times 0 = 0 \end{aligned}$$

$$\lim_{x \rightarrow 0} \left(\frac{e^x - 1}{x} \right) = 1 \text{ و } \lim_{x \rightarrow \infty} \left(\frac{x^2 + 5x + 4}{x^2 - 3x + 7} \right)^x = \lim_{x \rightarrow \infty} \left(1 + \frac{8x - 3}{x^2 - 3x + 7} \right)^x$$

$$\lim_{x \rightarrow e} \frac{\ln x - 1}{(x - e)} = \lim_{x \rightarrow e} \frac{\ln x - 1}{e \left(\frac{x}{e} - 1 \right)} = e^{\lim_{x \rightarrow \infty} \left(\frac{8x - 3}{x^2 - 3x + 7} \right) \cdot x} = e^8$$

$$\lim_{x \rightarrow \infty} \left(\frac{x^2 + 5x + 4}{x^2 - 3x + 7} \right)^x = e^8 \text{ إذن:}$$

$$\lim_{x \rightarrow e} \frac{\ln \left(\frac{x}{e} \right)}{e \left(\frac{x}{e} - 1 \right)} = \frac{1}{e} \cdot 1 = \frac{1}{e}$$

$$\lim_{x \rightarrow e} \frac{\ln x - 1}{(x - e)} = \frac{1}{e} \text{ إذن:}$$

$$\lim_{x \rightarrow 1} \left(\frac{\ln x}{x - 1} \right) = 1 \text{ نعلم أن:}$$

$$\text{بوضع : } m = \frac{1}{n} \text{ نجد:}$$

$$\lim_{x \rightarrow \frac{\pi}{3}} \frac{\sin x - \sqrt{3} \cos x}{4x - \frac{x\pi}{3}} \text{ نهاية :48}$$

$$\tan x - \tan y = \frac{\sin(x - y)}{\cos x \cdot \cos y} \text{ نعلم أن:}$$

$$\lim_{x \rightarrow \frac{\pi}{3}} \frac{\sin x - \sqrt{3} \cos x}{4x - \frac{4\pi}{3}}$$

$$= \lim_{x \rightarrow \frac{\pi}{3}} \cos(x) \times \lim_{x \rightarrow \frac{\pi}{3}} \frac{\tan x - \sqrt{3}}{4x - \frac{4\pi}{3}}$$

$$= \frac{1}{2} \times \lim_{x \rightarrow \frac{\pi}{3}} \frac{\tan x - \tan \frac{\pi}{3}}{4 \left(x - \frac{\pi}{3} \right)}$$

$$\frac{\sin \left(x - \frac{\pi}{3} \right)}{\cos x \cdot \cos \frac{\pi}{3}} = \frac{1}{2} \times \lim_{x \rightarrow \frac{\pi}{3}} \frac{\sin \left(x - \frac{\pi}{3} \right)}{4 \left(x - \frac{\pi}{3} \right)}$$

$$\lim_{m \rightarrow \infty} \left(\cos \left(\frac{x}{m} \right) \right)^m = \lim_{n \rightarrow 0} \left(\cos(nx) \right)^{\frac{1}{n}}$$

$$= e^{\lim_{n \rightarrow 0} \frac{\ln(\cos(nx))}{n}}$$

$$= e^{\lim_{n \rightarrow 0} \frac{\ln(\cos(nx))}{\cos(nx) - 1} \times \frac{\cos(nx) - 1}{n}}$$

$$= e^{\lim_{n \rightarrow 0} \frac{-2\sin^2(nx)}{n}}$$

$$= e^{\lim_{n \rightarrow 0} \frac{\sin(nx)}{n} \times (-2\sin(nx))}$$

$$= e^0 = 1$$

$$\lim_{x \rightarrow \infty} \left(\cos \left(\frac{x}{m} \right) \right)^m = 1 \text{ إذن:}$$

$$\lim_{x \rightarrow e} \frac{\ln x - 1}{(x - e)} \text{ نهاية :47}$$

$$\lim_{x \rightarrow 1} \left(\frac{\ln x}{x - 1} \right) = 1$$

$$\lim_{x \rightarrow 0} \left(\frac{\ln(x+1)}{x} \right) = 1 \text{ نعلم أن:}$$

ملاحظة 

طريقة حل النهايات ليست وحيدة

تجدون هذا الهمف



$$\begin{aligned} & \frac{\sin\left(x - \frac{\pi}{3}\right)}{\cos x \cdot \cos \frac{\pi}{3}} \\ &= \frac{1}{2} \times \lim_{x \rightarrow \frac{\pi}{3}} \frac{\sin\left(x - \frac{\pi}{3}\right)}{4\left(x - \frac{\pi}{3}\right)} \\ &= \frac{1}{2} \times \lim_{x \rightarrow \frac{\pi}{3}} \frac{\sin\left(x - \frac{\pi}{3}\right)}{4\left(x - \frac{\pi}{3}\right)} \times \frac{1}{\cos x \cdot \cos \frac{\pi}{3}} \\ &= \frac{1}{2} \end{aligned}$$

اذن: $\lim_{x \rightarrow \frac{\pi}{3}} \frac{\sin x - \sqrt{3} \cos x}{4x - \frac{x\pi}{3}} = \frac{1}{2}$

نهاية 49: $\lim_{x \rightarrow a} \frac{x^m - a^m}{x^n - a^n}$

$$\lim_{x \rightarrow a} \frac{x^m - a^m}{x^n - a^n} = \frac{m}{n} (a)^{m-n}$$

نهاية 50: $\lim_{x \rightarrow 0} \frac{1 - \sqrt[6]{\cos 2x}}{x^2}$

نعلم أن: $\lim_{x \rightarrow a} \frac{x^m - a^m}{x^n - a^n} = \frac{m}{n} (a)^{m-n}$

$$\begin{aligned} & \lim_{x \rightarrow 0} \frac{1 - \sqrt[6]{\cos 2x}}{x^2} \\ &= \lim_{x \rightarrow 0} \frac{1 - \sqrt[6]{\cos 2x}}{1 - \cos 2x} \times \frac{1 - \cos 2x}{x^2} \\ &= \lim_{x \rightarrow 0} \frac{1 - (\cos 2x)^{\frac{1}{6}}}{1 - \cos 2x} \times \frac{1 - \cos 2x}{x^2} \\ &= \frac{1}{n} \cdot (1)^{\frac{1}{6}-1} \cdot \lim_{x \rightarrow 0} \frac{\sin^2 x}{x^2} = \frac{1}{3} \end{aligned}$$

اذن: $\lim_{x \rightarrow 0} \frac{1 - \sqrt[6]{\cos 2x}}{x^2} = \frac{1}{3}$